Modern portfolio theory (**MPT**) proposes how rational investors will use diversification to optimize their portfolios, and how a risky asset should be priced. The basic concepts of the theory are Markowitz diversification, the efficient frontier, capital asset pricing model, the alpha and beta coefficients, the Capital Market Line and the Securities Market Line.

For a two asset portfolio:-

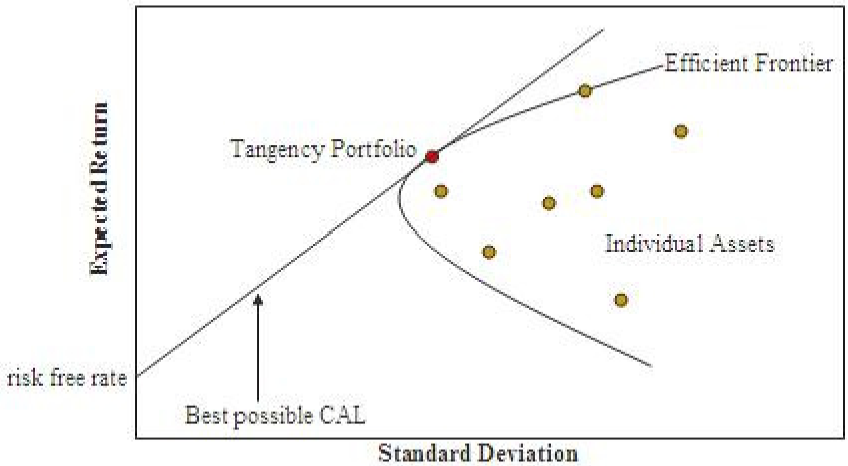
Portfolio return:

$$\mathrm{E}(R_p) = w_A \mathrm{E}(R_A) + (1 - w_A)\mathrm{E}(R_B) = w_A \mathrm{E}(R_A) + w_B \mathrm{E}(R_B)$$

Portfolio variance: $\sigma_p^2 = w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_Aw_B\rho_{AB}\sigma_A\sigma_B$

Correlation: As long as the correlation between the returns of two securities is less than +1, variances of returns of a portfolio is less than a weighted average of the individual variances of the portfolio securities. The lower the correlation between 2 securities the greater the diversification benefits;

The efficient frontier: the returns of most of the stocks have positive correlation (covariance's) with each other due to the similar effects of economy wide factors on the returns, though these correlations are less than perfect (less than +1), leading to risk reduction benefits of diversification.



All possible asset combination can be plotted in risk-return space, and the collection of all such possible portfolios defines a region in this space. **The line along the upper edge of this region is known as the *efficient frontier* (sometimes "the Markowitz frontier").** Combinations along this line represent portfolios (explicitly excluding the risk-free alternative) for **which there is lowest risk for a given level of return.**

The risk-free asset is the (hypothetical) asset which pays a risk-free rate. It is usually proxied by an investment in short-dated Government securities. The risk-free asset has zero variance in returns (hence is risk-free); it is also uncorrelated with any other asset (by definition: since its variance is zero).

As a result, when it is combined with any other asset, or portfolio of assets, the change in return and *also in risk* is linear.

Capital market line: When a risky portfolio is combined with some allocation to a risk – free asset, the resulting risk- return combinations will lie on a straight line between the two. This line is called the *Capital Market Line*. **All points along the *CML* have superior risk-return profiles to any portfolio on the efficient frontier**. All of these portfolios represent the highest possible Sharpe ratio. One can prove that the CML is the optimal CAL and that its equation is

$$\mathrm{CML} : E(r_C) = r_F + \sigma_C \frac{E(r_M) - r_F}{\sigma_M}.$$

Where In this formula *M* is the risky portfolio, *F* is the riskless portfolio, and *C* is a combination of portfolios *M* and *F*

Beta: The part of an individual security's risk that arises because of the positive covariance of the security's return with overall market return's is called systematic risk **(non-diversifiable risk- or beta).** The part of the volatility of a single security's return that is uncorrelated with the volatility of the market portfolio is that security's diversifiable risk or non systematic risk. (the risk reduction in efficient portfolios comes from reducing the diversifiable risk) . An asset with a beta of 0 means that its price is not at all correlated with the market; that asset is independent. A positive beta means that the asset generally follows the market. A negative beta shows that the asset inversely follows the market; the asset generally decreases in value if the market goes up.

The formula for the Beta of an asset within a portfolio is $\beta_a = \frac{\mathrm{Cov}(r_a, r_p)}{\mathrm{Var}(r_p)}$

CAPM: The CAPM formula takes into account the asset's sensitivity to only systematic risk – represented by Beta, (Since the non systematic risk can be avoided by efficient diversification, there is no added expected return for bearing nonsystematic return on the market) as well as the expected return of the market and the expected return of a theoretical risk-free asset.

Capital Asset Pricing Model (CAPM): $E(R_i) = R_f + \beta_{im}(E(R_m) - R_f).$

Where:

$E(R_i)$ is the expected return on the capital asset ,

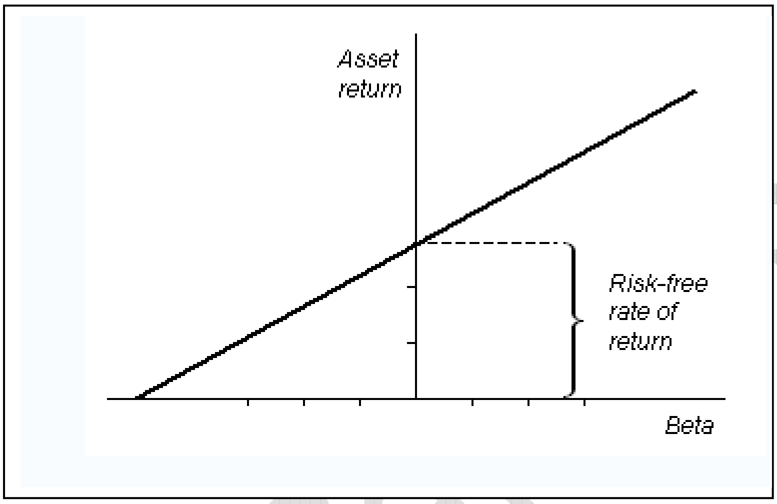
R_f is the risk-free rate of interest (the intercept)

β_{im} (the *beta coefficient*) the sensitivity of the asset returns to market returns, or also $\beta_{im} = \frac{\mathrm{Cov}(R_i, R_m)}{\mathrm{Var}(R_m)}$,

$E(R_m)$ is the expected return of the market

$E(R_m) - R_f$ is sometimes known as the *market premium* or *risk premium* (the slope of the line)

Required return %



Security Market line: The result of the CAPM model is represented by the SML as shown in the above figure. It tells us the requires return an investor should earn in the marketplace for any level of Beta risk.

Important Concepts

1. **The Sharpe ratio** or Sharpe index or Sharpe measure or reward-to-variability ratio is a measure of the excess return (or Risk Premium) divided by per unit of total risk in an investment asset or a trading strategy.

$$S = \frac{E[R - R_f]}{\sigma} = \frac{E[R - R_f]}{\sqrt{\mathrm{var}[R - R_f]}},$$

where *R* is the asset return, R_f is the return on a benchmark asset, such as the risk free rate of return, $E[R - R_f]$ is the expected value of the excess of the asset return over the benchmark return, and σ is the standard deviation of the excess return

2. **The Treynor ratio** is a measurement of the returns earned in excess of that which could have been earned on a riskless investment (i.e. Treasury Bill) divided by per unit of market risk in an investment asset.

The Treynor ratio (sometimes called reward-to-volatility ratio) relates excess return over the risk-free rate to the additional risk taken; however systematic risk instead of total risk is used. The higher the Treynor ratio, the better the performance under analysis.

$$T = \frac{r_p - r_f}{\beta}$$

where

$T \equiv$ Treynor ratio,

$r_p \equiv$ portfolio return,

$r_f \equiv$ risk free rate

$\beta \equiv$ portfolio beta

The treynor & sharpe ratio measure the relative performance of various portfolios on risk adjusted basis. Alpha measure an absolute performance on risk – adjusted basis.

3. **Alpha** is a risk-adjusted measure of the so-called "excess return" on an investment. It is a common measure of assessing an active manager's performance as it is the return in excess of a benchmark index.The difference between the fair and actually expected rates of return on a stock is called the stock's alpha.

   The **alpha coefficient** (α_i) is a parameter in the capital asset pricing model. In fact it is the intercept of the **Security Characteristic Line** (**SCL**). One can prove that in an efficient market, the expected value of the **alpha coefficient** equals the return of the risk free asset: $E(\alpha_i) = r_f$.

   Therefore the alpha coefficient can be used to determine whether an investment manager or firm has created economic value:

   $\alpha_i < r_f$: the manager or firm has destroyed value

   $\alpha_i = r_f$: the manager or firm has neither created nor destroyed value

   $\alpha_i > r_f$: the manager or firm has created value

   The difference $\alpha_i - r_f$ is called Jensen's alpha.

4. **Jensen's alpha** (or **Jensen's Performance Index**, **ex-post alpha**) is used to determine the excess return of a stock, other security, or portfolio over the security's required rate of return as determined by the Capital Asset Pricing Model.

5. The **Information Ratio** measures the excess return of an investment manager divided by the amount of risk the manager takes relative to a benchmark. It is used in the analysis of performance of mutual funds, hedge funds, etc. Specifically, the information ratio is defined as excess return divided by tracking error. Excess return is the amount of performance over or under a given benchmark index. Thus, excess return can be positive or negative. Tracking error is the standard deviation of the excess return.